

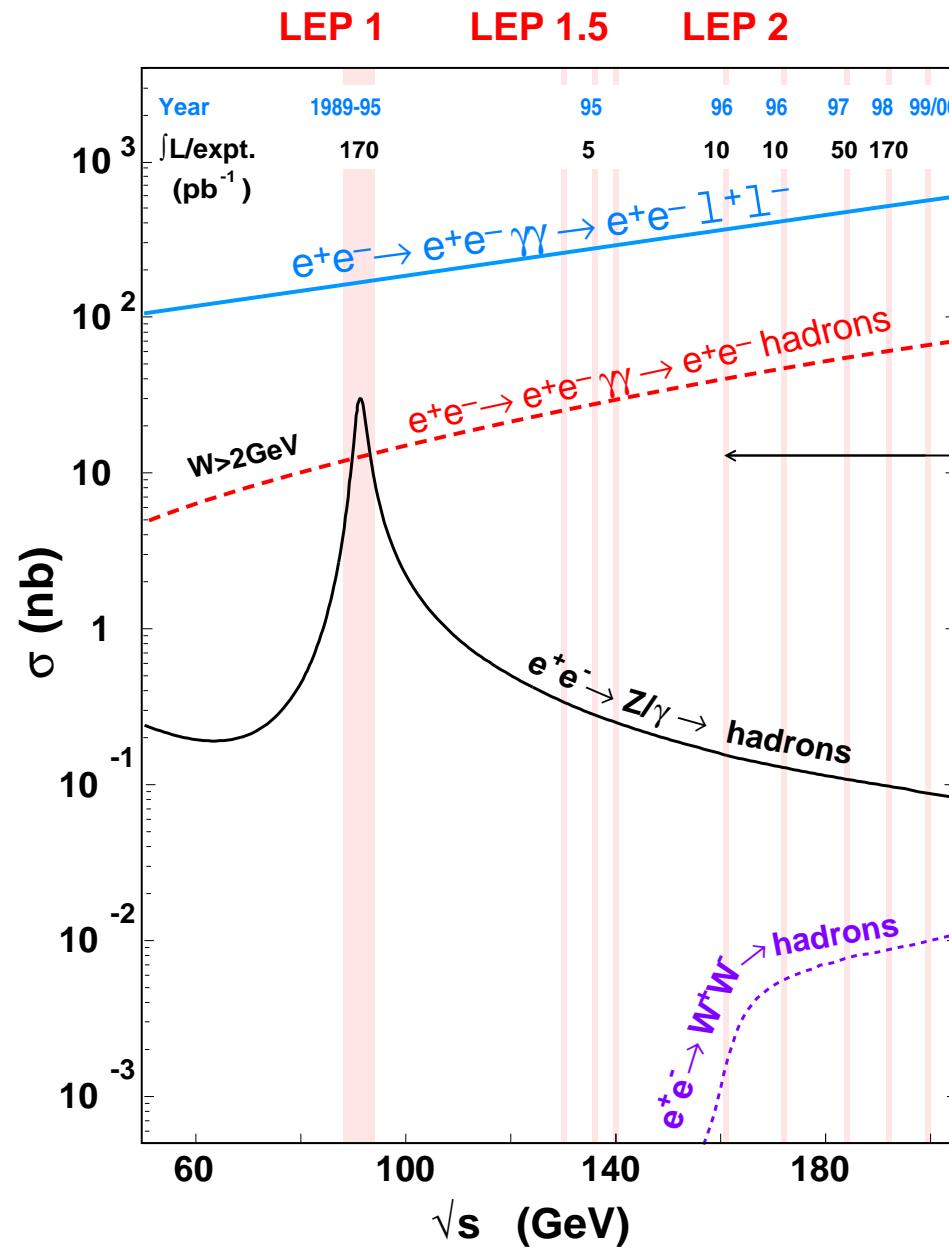
# **QCD RADIATIVE CORRECTIONS**

TO

## **$\gamma^* \gamma^* \rightarrow \text{HADRONS}$**

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# THE LEP CROSS SECTIONS



**GOAL**

to analyse the QCD dynamics in the  $s \gg |t|$  limit:  
the high energy limit (HEL)

**FACT**

in HEL the scattering processes are dominated by  
sub-processes with gluon exchange in the  $t$  channel

**BFKL**

theory resums multiple gluon radiation out of  
the gluon exchanged in the  $t$  channel

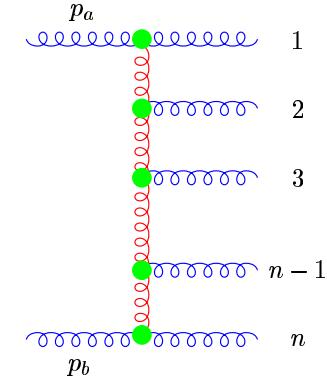
**PHENOM.**

Process-dependent questions:

- does a fixed-order expansion in  $\alpha_s$  suffice to describe the data ?
- can the data be described in terms of other, e.g. Sudakov,  
resummations ?
- in phase space, where do sub-processes with gluon exchange in the  
 $t$  channel dominate over the other sub-processes ?

# BFKL RESUMMATION

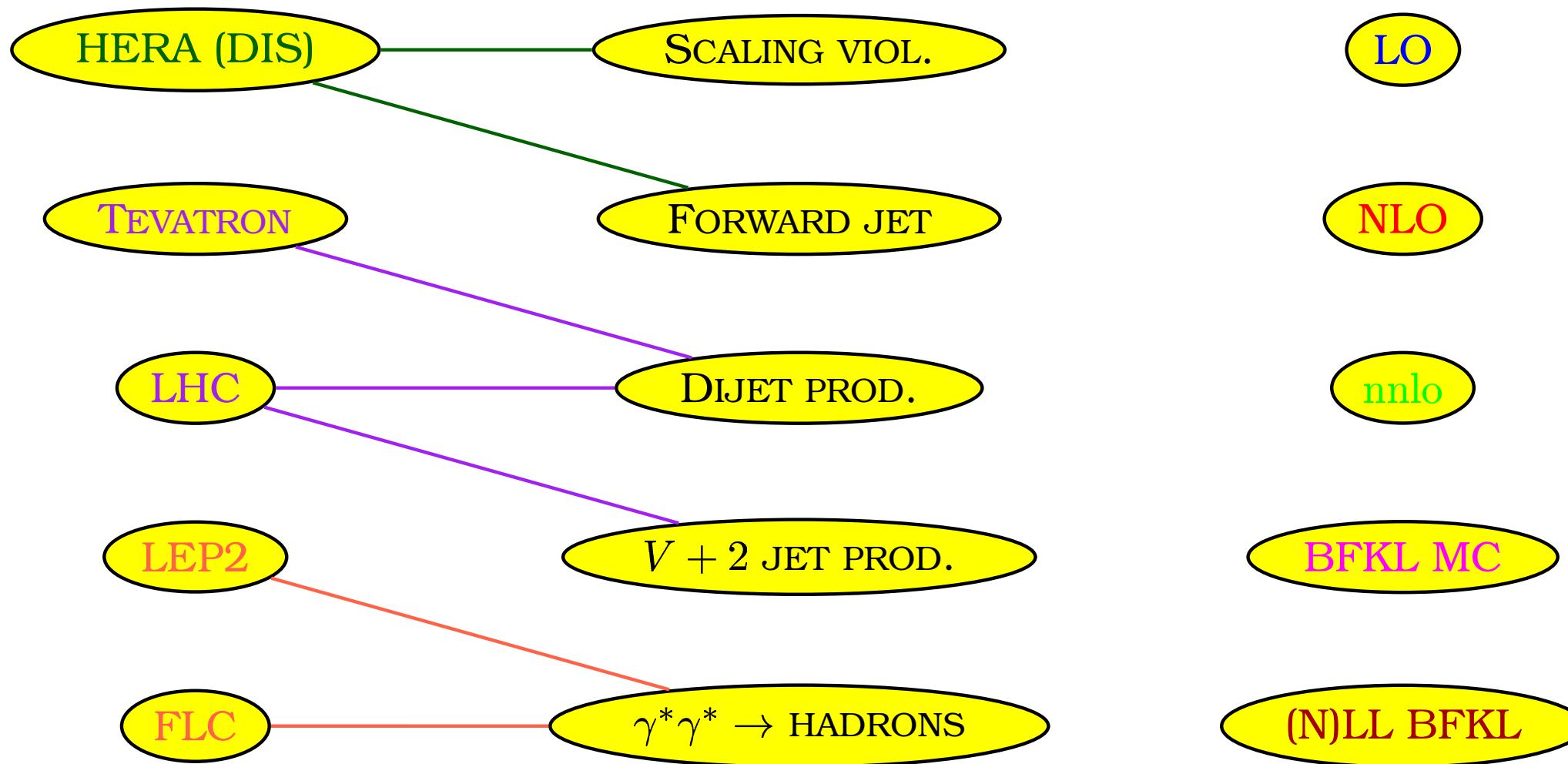
- ☛ in any scattering process with  $s \gg |t|$  gluon exchange in the  $t$  channel dominates



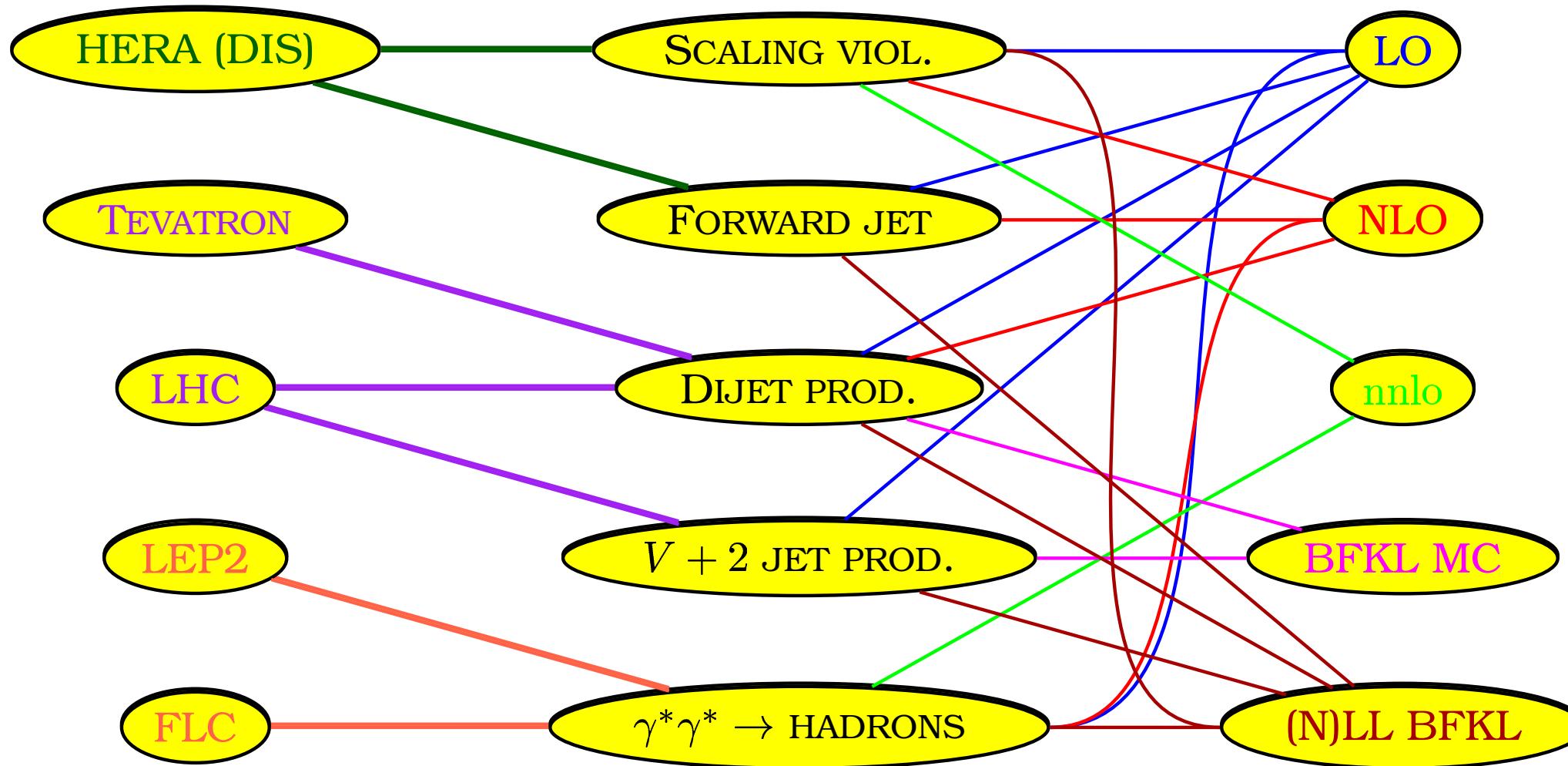
- ☛ BFKL is a resummation of multiple gluon radiation out of the gluon exchanged in the  $t$  channel

- ☛ for  $s \gg |t|$  BFKL resums the Leading Log (and Next-to-Leading Log) contributions, in  $\log(s/t)$ , of the radiative corrections to the gluon propagator in the  $t$  channel, to all orders in  $\alpha_s$
- ☛ the LL terms are obtained in the approximation of strong rapidity ordering ( $y_1 \gg y_2 \gg \dots \gg y_n$ ) and no  $k_t$  ordering of the emitted gluons
- ☛ the NLL terms are universal
- ☛ the resummation yields a 2-dim integral equation for the evolution of the gluon propagator in the  $t$  channel

# STATUS OF BFKL ANALYSES

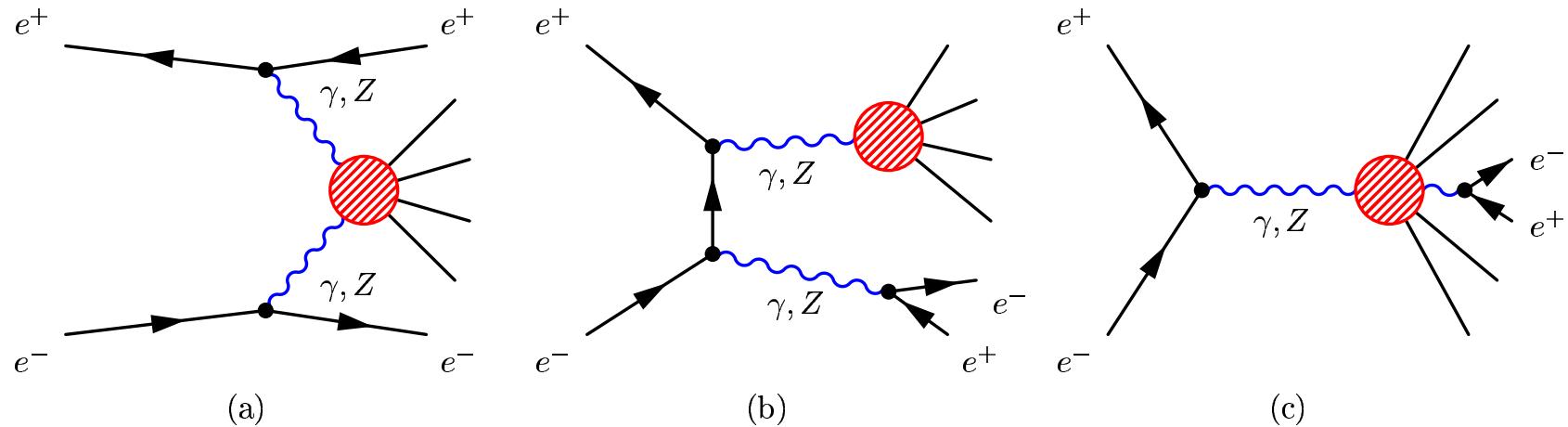


# STATUS OF **BFKL** ANALYSES



## $e^+e^- \rightarrow e^+e^-$ HADRONS

Several ( $> 100$ ) diagrams contribute to  $e^+e^- \rightarrow e^+e^-$  hadrons



**L3 & OPAL:** small scattering angles of the outgoing leptons make:

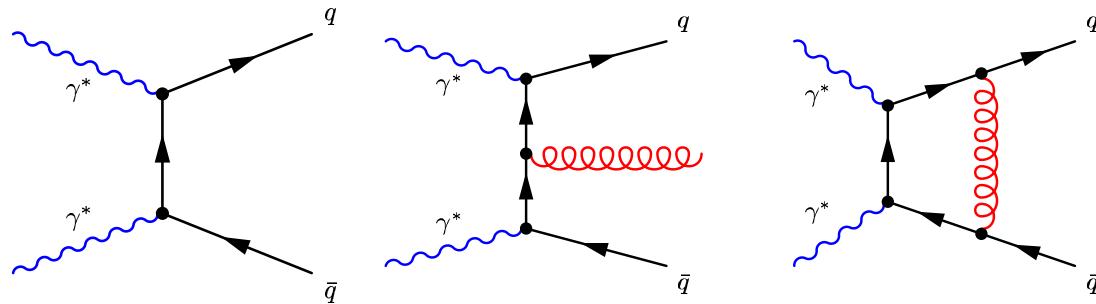
- ☞ annihilation processes negligible
- ☞ photon virtualities small  $\rightarrow Z$  contribution negligible

thus, only diagrams of type (a) survive:

$$e^+e^- \rightarrow e^+e^- + \underbrace{\gamma^*\gamma^*}_{\text{hadrons}}$$

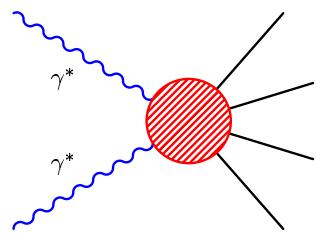
# $\gamma^* \gamma^* \rightarrow \text{HADRONS}$

The fixed order expansion in  $\alpha_s$

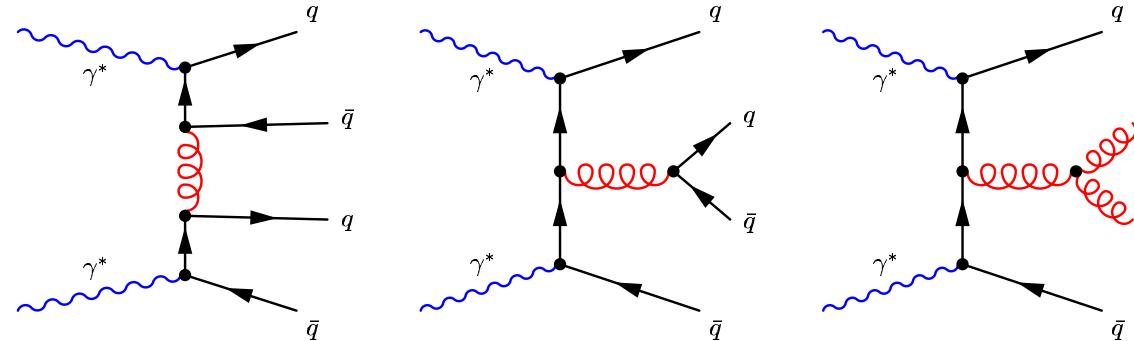


LO

Cacciari, Frixione, Trocsanyi, VDD, hep-ph/0011368



=



BFKL (Born)

NNLO (4-partons)

Maltoni, Trocsanyi, VDD, hep-ph/0202237

## THEORETICAL FRAMEWORK

$\gamma^*\gamma^*$  cross section as fixed-order expansion + resummation

$$\sigma_{\gamma^*\gamma^*} \sim \sum_{j=0}^{\infty} a_{0j} \alpha_s^j + a_1 \alpha_s^2 \sum_{j=0}^{\infty} \left( \alpha_s \log \left( \frac{W^2}{\mu_w^2} \right) \right)^j + \dots$$

- 👉  $W$ : hadronic energy       $\mu_w$ : transverse energy scale
- 👉  $2^{nd}$  sum collects terms which feature **only gluon** exchange in the  $t$  channel, and resums leading log (**LL**) corrections
- 👉 ellipses refer to log corrections beyond **LL**
- 👉  $1^{st}$  sum is a fixed-order expansion in  $\alpha_s$  starting at  $\mathcal{O}(\alpha_s^0)$ , and collects the contributions which **do not** feature **only gluon** exchange in the  $t$  channel
- 👉  $a_{0j}$  behave like  $1/W^2$  (or eventually like  $1/(W\mu_w)$ )
- 👉  $a_{00}$  is **LO** term;  $a_{01}$  is **NLO** term;  $a_{02} + a_1(j=0)$  is **NNLO** term
- 👉  $a_1$  behaves like  $1/\mu_w^2$

## PHASE SPACE

factorises into leptonic & hadronic parts

## KINEMATICS

we use the variable  $Y = \log \frac{y_1 y_2 s}{\sqrt{Q_1^2 Q_2^2}}$

$y_i$   $\propto$  the light-cone momentum fraction of the virtual photons

$$y_i = \frac{q_i^0 + q_i^3}{\sqrt{s}} = 1 - \frac{2E_i}{\sqrt{s}} \cos^2 \frac{\theta_i}{2}, \quad i = 1, 2$$

$E_i, \theta_i$ : energies and scattering angles of the leptons in the  $e^+e^-$  frame

\* IN HEL  $y_1 y_2 s \simeq W^2$

→  $Y \simeq \log \frac{W^2}{\sqrt{Q_1^2 Q_2^2}}$  becomes BFKL-like variable

## NLO CALCULATION

- we used a general-purpose NLO partonic Monte Carlo (subtraction)
- we took the massless limit of the outgoing quarks

## $\mu_R$ SCALE

the calculation

\* is LO in  $\alpha_{\text{em}}$

$\alpha_{\text{em}}(\mu_R^2)$ : we choose  $\alpha_{\text{em}}^2(Q_1^2)\alpha_{\text{em}}^2(Q_2^2)$  (one-loop  $\overline{\text{MS}}$  running)

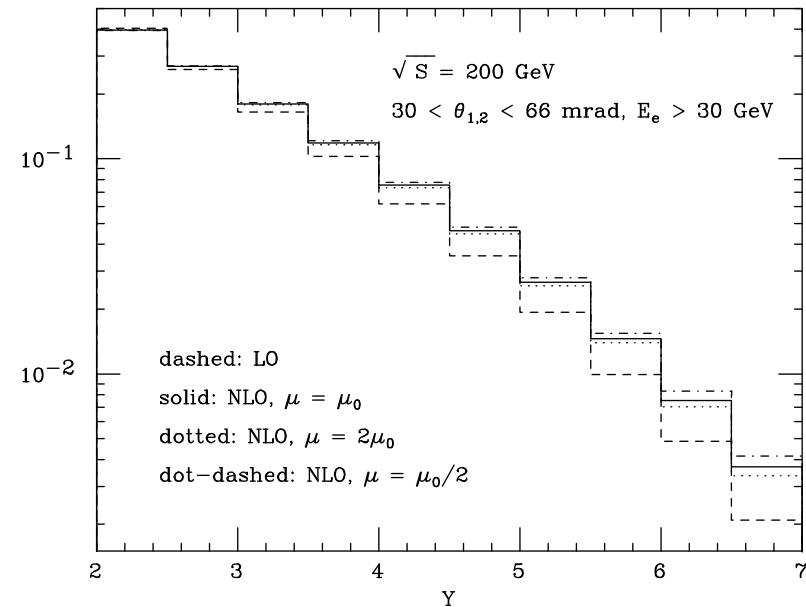
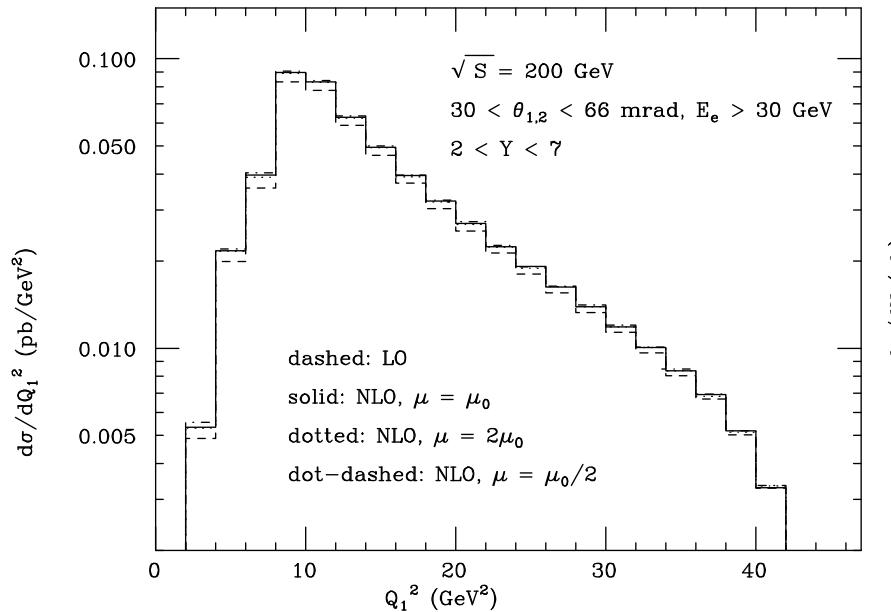
\* has  $\alpha_s$  occurring first at NLO

$$\alpha_s(\mu_R^2): \quad \mu_0^2 = \frac{Q_1^2 + Q_2^2}{2} + \left( \frac{\sum_i p_{i\perp}}{2} \right)^2 \quad i=1,2,3$$

$p_{i\perp}$ : transverse momenta in the  $\gamma^*\gamma^*$  frame

## NLO SCALE UNCERTAINTIES

- ☛  $Q^2$  &  $Y$  distributions
- ☛ we varied  $\frac{\mu_0}{2} < \mu_R < 2\mu_0$



L3 CUTS

$E_i \geq 30$  GeV     $30 \leq \theta_i \leq 66$  mrad     $2 \leq Y \leq 7$

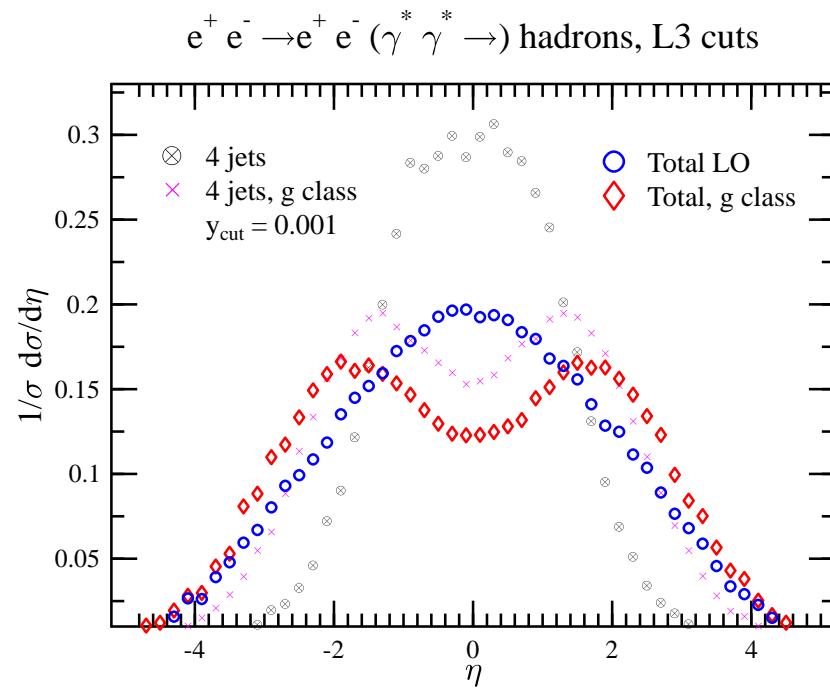
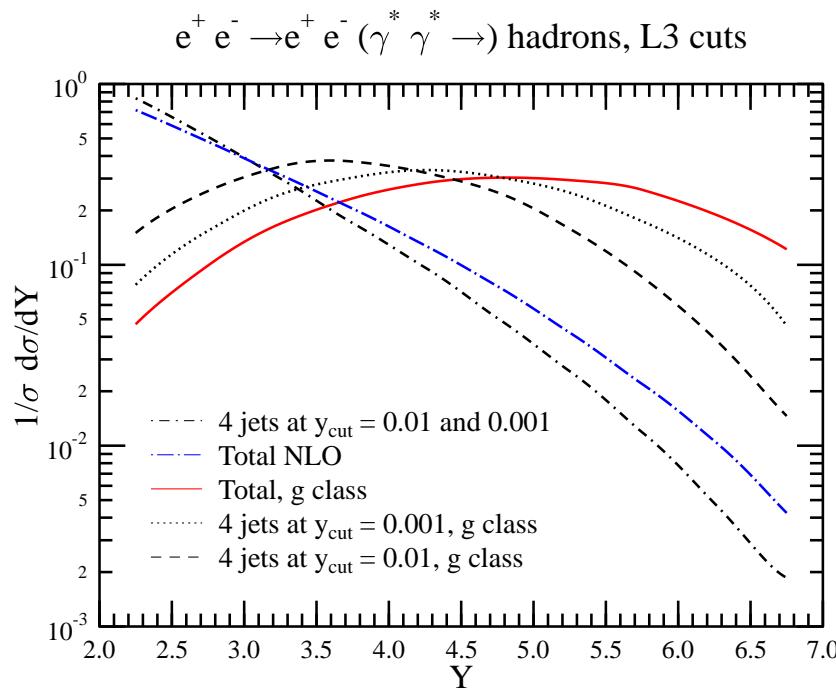
L3 Collaboration '99

- ☛ uncertainty related to  $\mu$  is smaller than NLO corrections
- ☛ effect of NLO corrections is small, except at large  $Y$ , where they induce a 50% increase

# NNLO 4-PARTON FINAL STATES

diagrams with exchange in the  $t$  channel of a

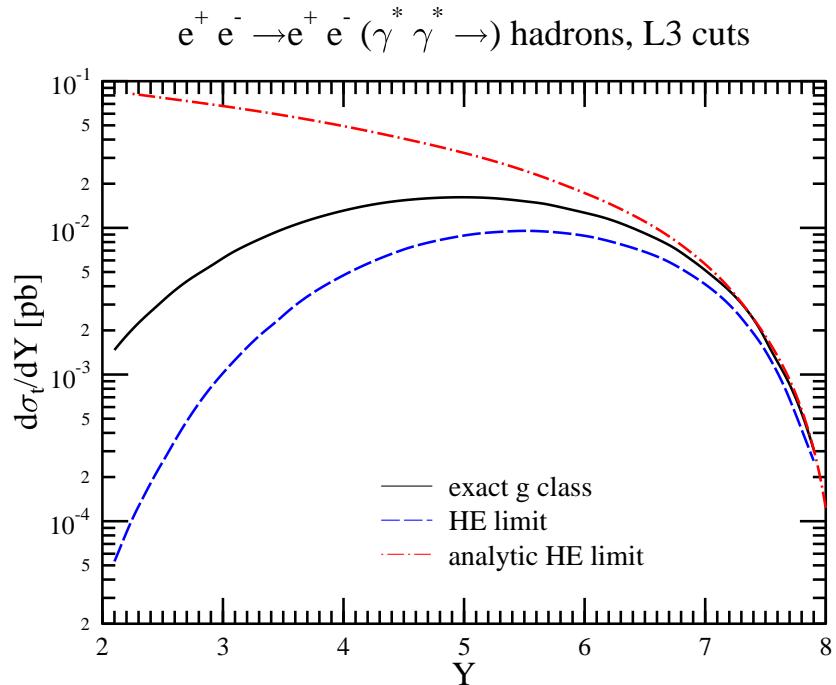
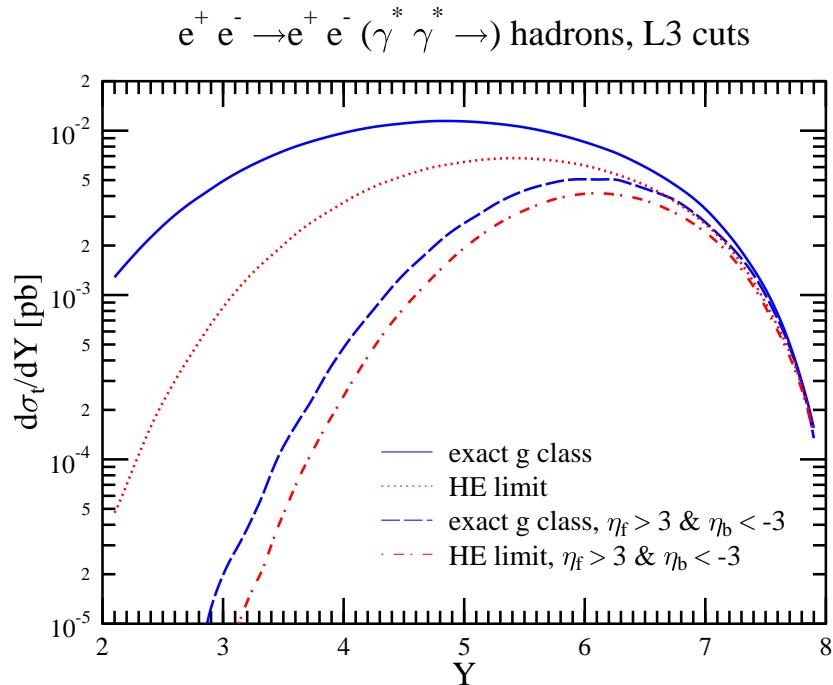
- \* gluon ( $g$  class) are gauge invariant & infrared finite
- \* quark ( $f$  class) display infrared divergences



- ❖ the  $g$  class and  $f$  class have different shapes in the  $Y$  and  $\eta$  distributions: the  $g$  class contributes more at large  $Y$  and its quarks are more forward

# THE HIGH ENERGY LIMIT (HEL)

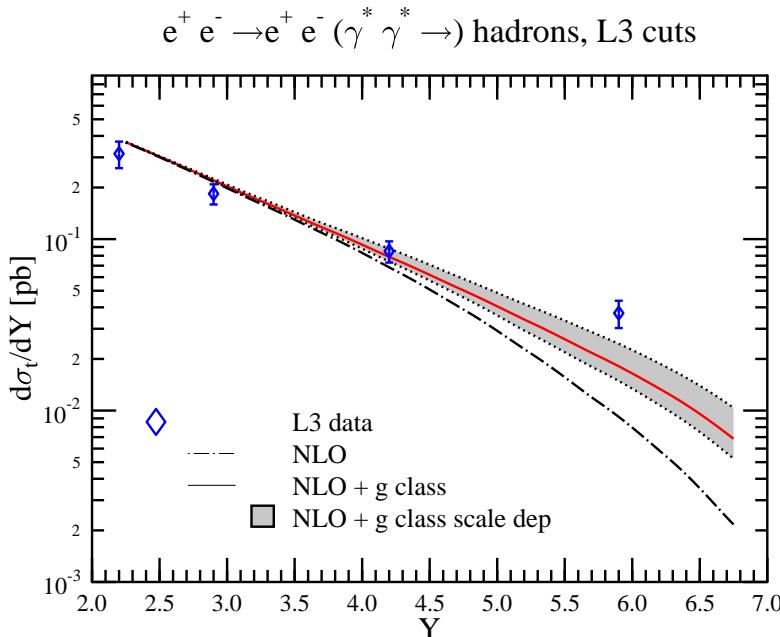
- \* in HEL the scattering amplitude of the *g* class and the 4-quark phase space factorise into two impact factors connected by the *t* channel gluon



- HEL yields errors smaller than 20% only for  $Y > 7$
- requiring the quarks forward underestimates exact *g* class
- analytic HEL is much larger than exact *g* class

# PHENOMENOLOGY

## L3 data versus NLO or NLO + $g$ class



$\Delta W_{\gamma\gamma}$ (GeV)	L3 data $d\sigma_e^e/dW_{\gamma\gamma}$ (pb/GeV)	NLO $d\sigma_{ee}/dW_{\gamma\gamma}$ (pb/GeV)	NLO + $g$ class $d\sigma_e^e/dW_{\gamma\gamma}$ (pb/GeV)
5–10	$0.07470 \pm 0.0096 \pm 0.0067$	$0.0883^{+0.0004}_{-0.0027}$	$0.0885^{+0.0003}_{-0.0027}$
10–20	$0.02630 \pm 0.0024 \pm 0.0024$	$0.0300^{+0.0001}_{-0.0001}$	$0.0305^{+0.0003}_{-0.0002}$
20–40	$0.00620 \pm 0.0007 \pm 0.0006$	$0.0057^{+0.0001}_{-0.0003}$	$0.0064^{+0.0006}_{-0.0003}$
40–100	$0.00140 \pm 0.0002 \pm 0.0001$	$0.0004^{+0.0001}_{-0.0000}$	$0.0007^{+0.0002}_{-0.0001}$
$\Delta Y$	L3 data $d\sigma_e^e/d\bar{Y}$ (pb)	NLO $d\sigma_{ee}/dY$ (pb)	NLO + $g$ class $d\sigma_e^e/dY$ (pb)
2.0–2.5	$0.3150 \pm 0.480 \pm 0.28$	$0.366^{+0.001}_{-0.001}$	$0.368^{+0.002}_{-0.002}$
2.5–3.5	$0.1840 \pm 0.180 \pm 0.17$	$0.203^{+0.002}_{-0.001}$	$0.208^{+0.004}_{-0.003}$
3.5–5.0	$0.0850 \pm 0.090 \pm 0.08$	$0.070^{+0.002}_{-0.002}$	$0.080^{+0.008}_{-0.005}$
5.0–7.0	$0.0370 \pm 0.060 \pm 0.03$	$0.010^{+0.001}_{-0.001}$	$0.018^{+0.006}_{-0.003}$

**L3 CUTS**

$E_i \geq 40$  GeV

$30 \leq \theta_i \leq 66$  mrad

$W \geq 5$  GeV

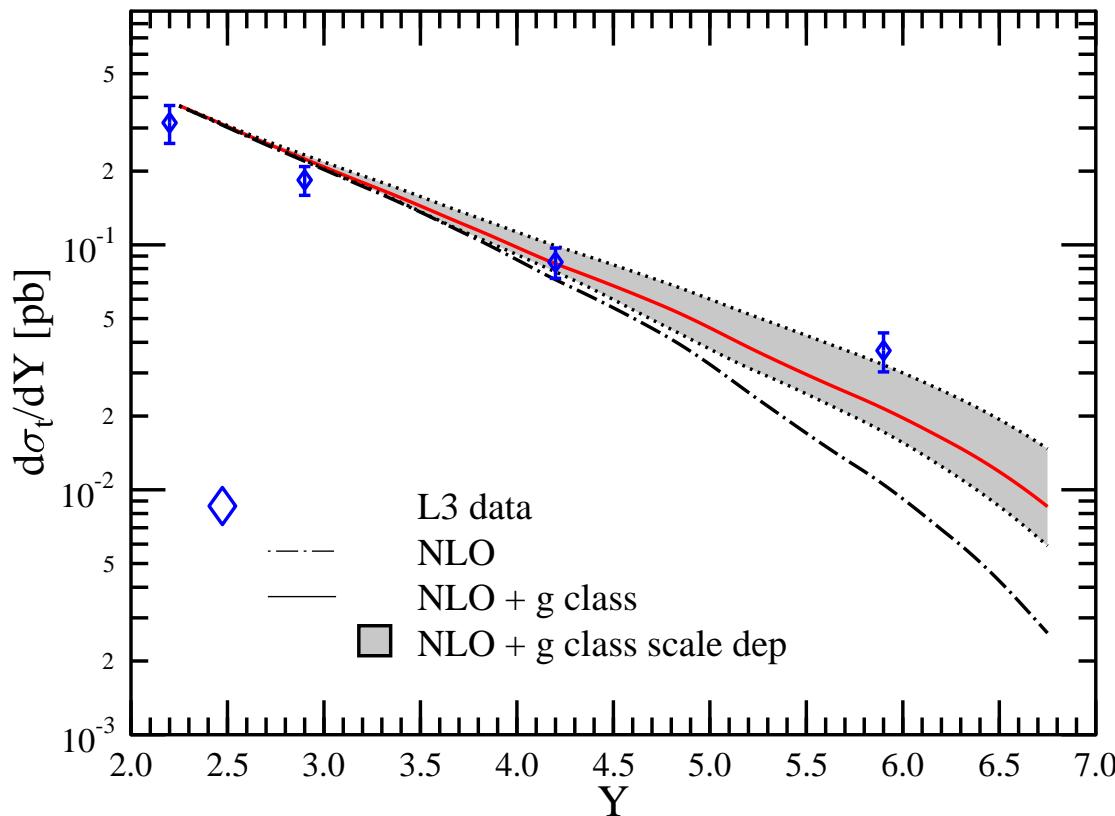
L3 Collaboration 2001

- \* shaded band: scale variation  $\frac{\mu_0}{2} < \mu_R < 2\mu_0$
- \* error bars on data: added statistical and systematic errors in quadrature
- \* slight excess of NLO over L3 data at small  $Y$  (REM: massless limit )
- \* NLO & NLO +  $g$  class underestimate L3 data at large  $Y$

# CAVEAT

\* however, if as a default scale for  $\mu_R$  use  $\mu_0^2 = \frac{Q_1^2 + Q_2^2}{2}$  , get

$e^+ e^- \rightarrow e^+ e^- (\gamma^* \gamma^* \rightarrow) \text{hadrons, L3 cuts}$



→ the *g* class contribution depends sizeably on  $\mu_R$  scale variations

## CONCLUSIONS

- ☞ NLO theory describes well the data, but for
  - \* slight excess at small  $Y$ , to be reduced by mass dependence
  - \* a slight deficit at large  $Y$
- ☞ 4-parton contributions of the NNLO theory
  - \* HEL is not accurate at LEP2 energies
  - \* 4-quark  $g$  class must be included exactly
- ☞ deficit at large  $Y$ 
  - \* NLO +  $g$  class reduces discrepancy between theory & L3 data
  - \* a full NNLO calculation (nowadays unfeasible) would be welcome
  - \* higher order corrections (BFKL ?) might be relevant